B.Sc Part – I

Relativistic Formula for composition of velocities

Consider two frames of reference S and S', S' is moving with velocity V along x-axis relative to S. Let (x, y, z, t) and (x', y', z', t') be co-ordinates of a moving point P is in S and S' respectively. The velocity components of P in S and S' are given by (u_x, u_y, u_z) and (u'_x, u'_y, u'_z) where

$$u_x = \frac{dx}{dt}, \quad u_y = \frac{dy}{dt}, \quad u_z = \frac{dz}{dt}$$

$$u'_x = \frac{dx'}{dt'}, \quad u'_y = \frac{dy'}{dt'}, \quad u'_z = \frac{dz'}{dt'}$$

Lorentz reverse transformations are

$$x = \beta(x' + Vt'),\ y = y',\ z = z',\ t = \beta\left(t' + \frac{Vx'}{c^2}\right)$$

where $$\beta = \frac{1}{\left(1 - \frac{V^2}{c^2}\right)^{1/2}}$$

Taking differentials

$$dx = \beta(dx' + V dt'),\ dy = dy',\ dz = dz',\ dt = \beta\left[dt' + \frac{V}{c^2}dx'\right]$$

$$u_x = \frac{dx}{dt} = \frac{\beta(dx' + V dt')}{\beta\left(dt' + \frac{V}{c^2}dx'\right)} = \frac{\frac{dx'}{dt'} + V}{\left(1 + \frac{V}{c^2}\frac{dx'}{dt'}\right)}$$

$$= \frac{u'_x + V}{\left(1 + \frac{V}{c^2}u'_x\right)}$$

$$u_y = \frac{dy}{dt} = \frac{dy'}{\beta\left(dt' + \frac{v}{c^2}dx'\right)} = \frac{dy'/dt'}{\beta\left(1 + \frac{v}{c^2}\frac{dx'}{dt'}\right)}$$

$$= u'_y / \beta\left(1 + \frac{v}{c^2}u'_x\right)$$

Similarly

$$u_z = u'_z / \beta\left(1 + \frac{v}{c^2}u'_x\right)$$

$$u_x = (u'_x + v) / \left(1 + \frac{v}{c^2}u'_x\right)$$

$$u_y = u'_y / \beta\left(1 + \frac{v}{c^2}u'_x\right)$$

$$u_z = u'_z / \beta\left(1 + \frac{v}{c^2}u'_x\right)$$

The velocity components (u_x, u_y, u_z) represent the result compounding the velocity (u'_x, u'_y, u'_z) and $(v, 0, 0)$

— x —